Fundamentals in Biophotonics

Photon /wave particle duality

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Exam modality change

Dear Madam,

Thank you for sending over the form. We confirm that we made the necessary change in IS-Academia with regards to the examination modality for BIO-443.

With our best regards,

EPFL

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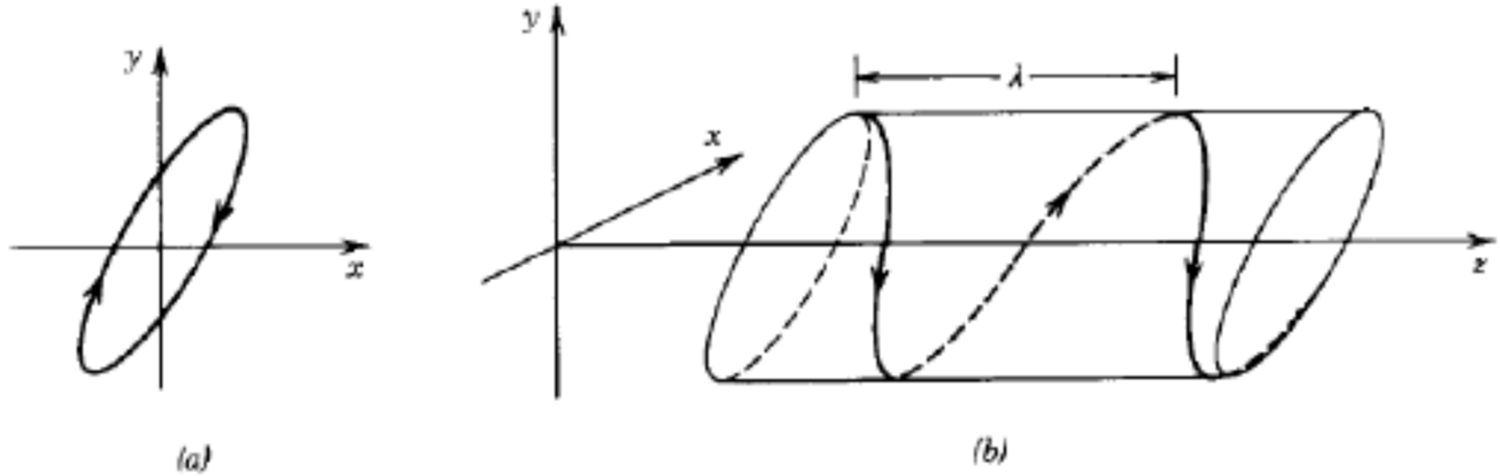
Polarization of Light



(*a*) Rotation of the endpoint of the electric-field vector in the *x-y* plane at a fixed position *z*. (*b*) Snapshot of the trajectory of the endpoint of the electric-field vector at a fixed time *t*.

At a fixed value of z, the tip of the electric-field vector rotates periodically in the x-y plane, tracing out this ellipse.

At a fixed time t, the locus of the tip of the electric-field vector follows a helical trajectory in space lying on the surface of an elliptical cylinder.!

Polarization of Light

- Consider a monochromatic plane wave of frequency ν, travelling in the ***z*** direction with velocity ***c***. The electric field lies in the x-y plane and is generally described by

$$\vec{\mathbf{E}}(z,t) = E_x\,\hat{\mathbf{x}} + E_y\,\hat{\mathbf{y}}$$

To describe the polarization of this wave, we trace the endpoint of the vector **E** (z,t) at each position z as a function of time

The components *Ex* and *Ey* are periodic functions of (t-z/c) oscillating at frequency ν

$$E_x = a_x \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \varphi_x\right]$$

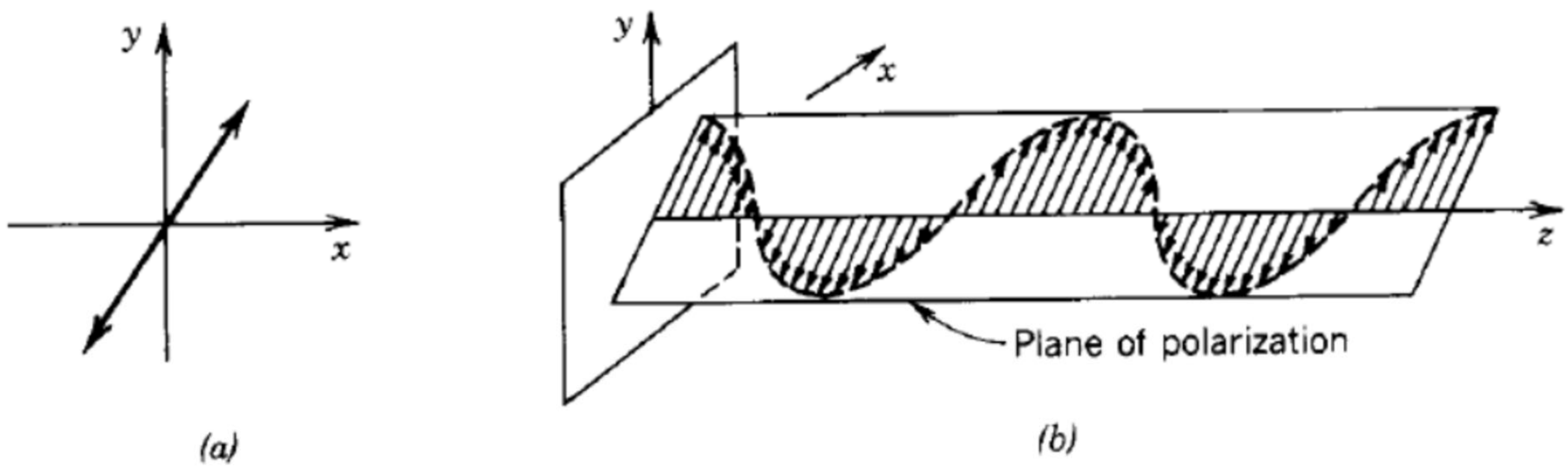
$$E_y = a_y \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \varphi_y\right]$$

$\varphi = \varphi_y - \varphi_x$ is the **phase difference**.

Linearly Polarized Light

- If one of the components vanishes (a_x=0, for example), the light is linearly polarized in the direction of the other component (the y direction). The wave is also linearly polarized if the phase difference ϕ=0 or , since

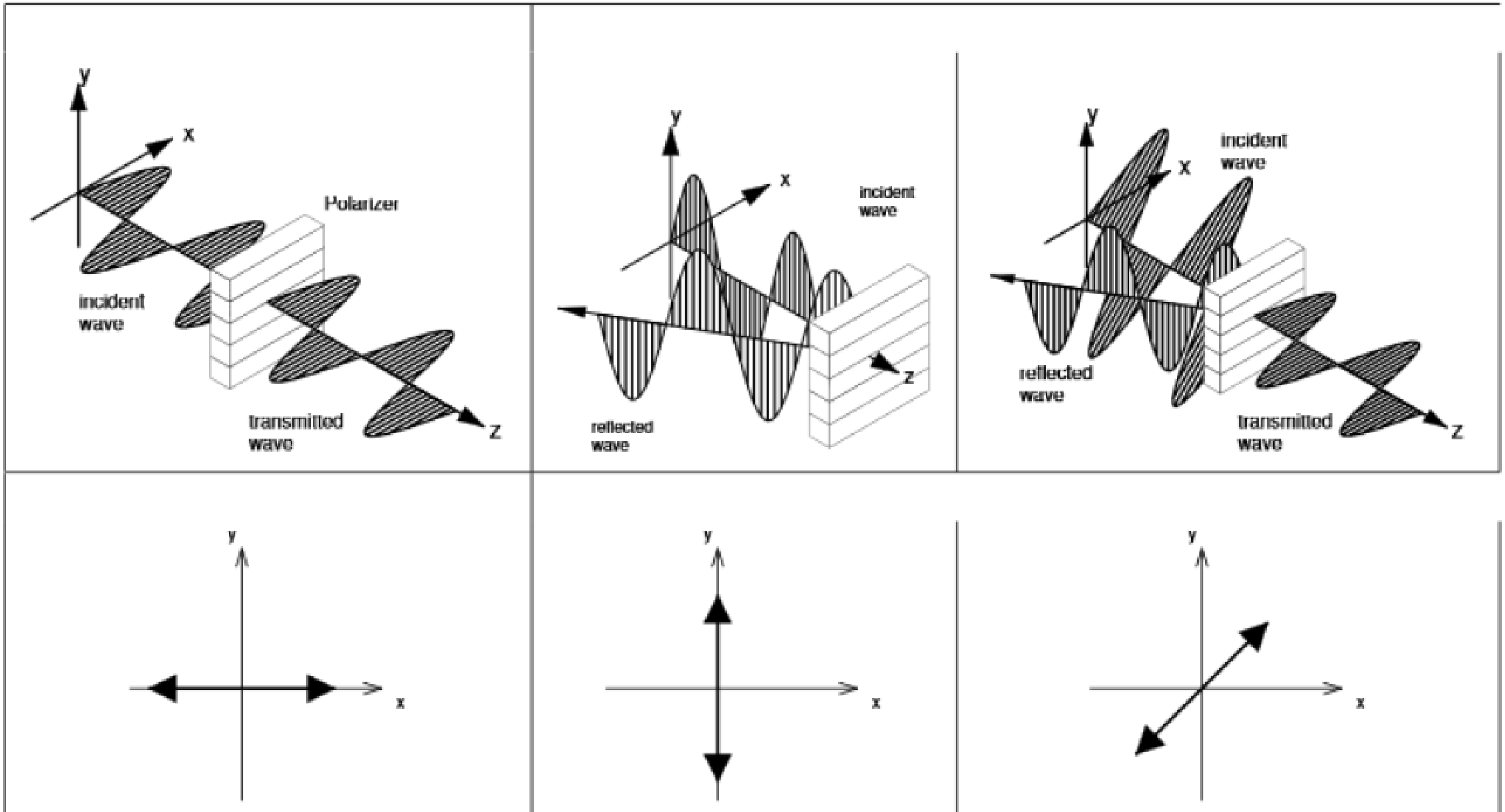
$$E_y = \pm (a_y / a_x) E_x$$



Linearly polarized light. (*a*) Time course at a fixed position *z*. (*b*) A snapshot (fixed time *t*).

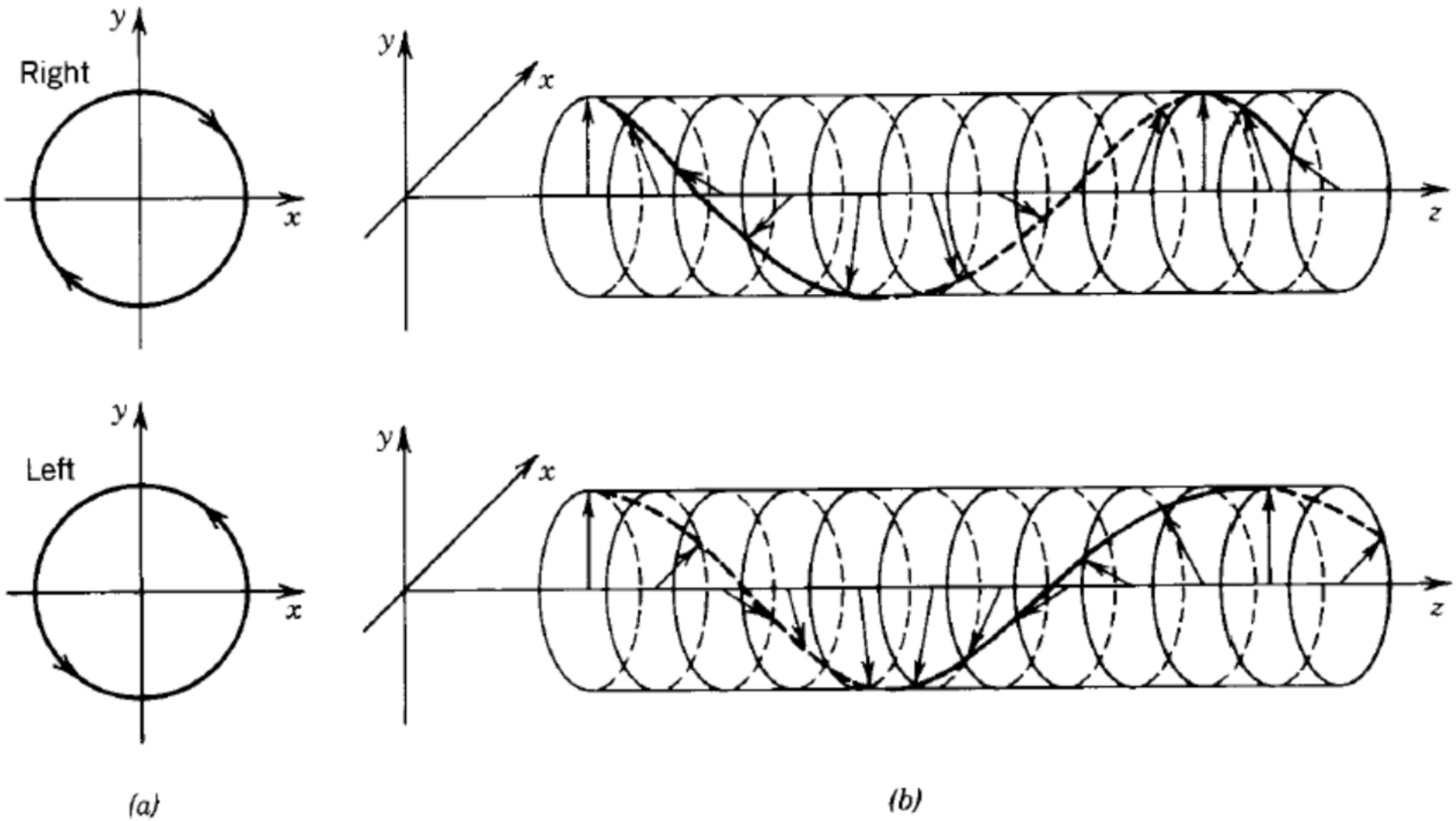
Linearly polarized light incident to a polarizer

- light is travelling in the positive *z*-direction, with angular frequency ω and wavevector $\mathbf{k} = (0,0,k)$, where the wavenumber $k = \omega/c$.



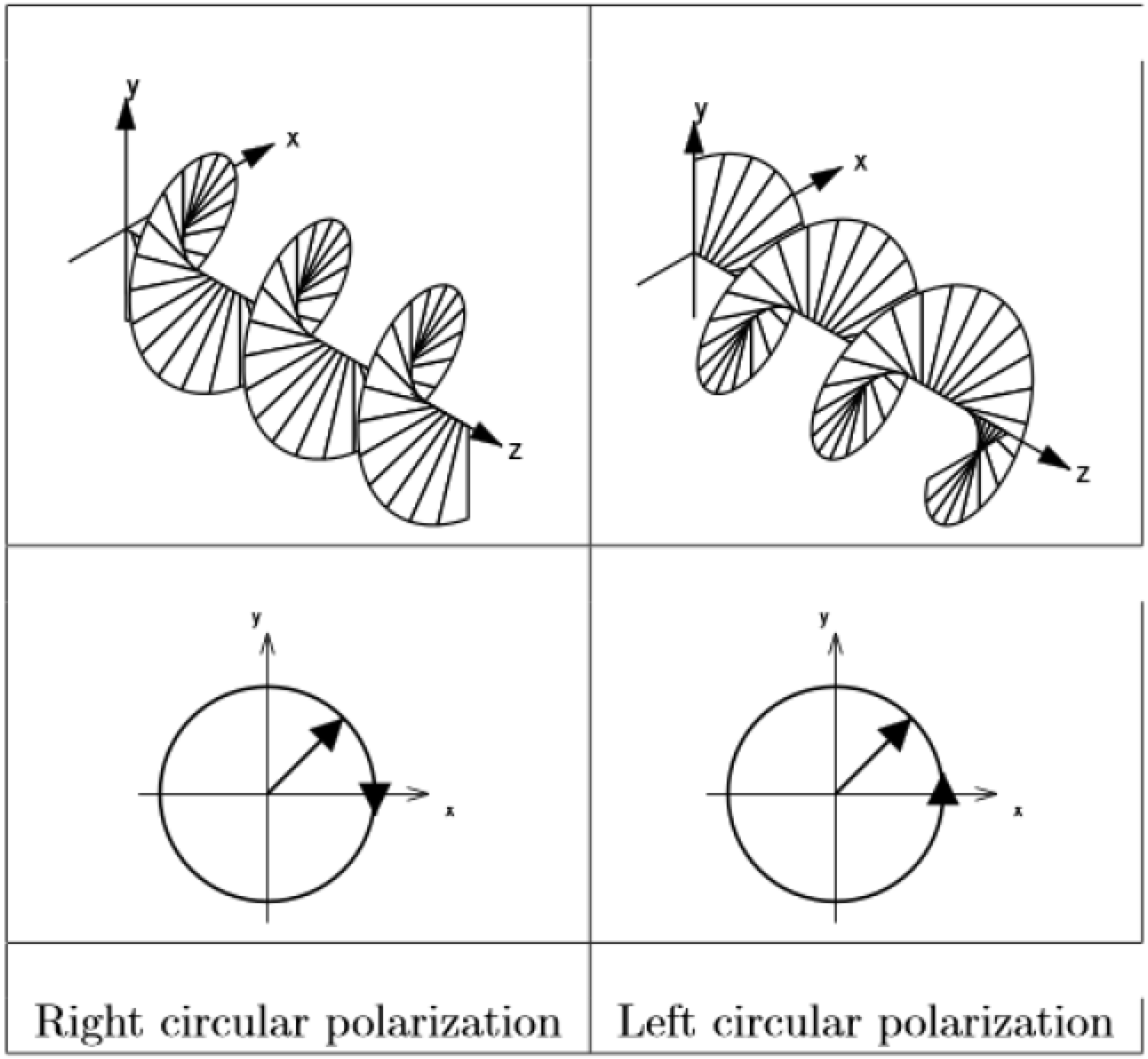
Horizontal polarization | Vertical Polarization | Arbitrary linear polarization

Circularly Polarized Light



Trajectories of the endpoint of the electric-field vector of a circularly polarized plane wave. (a) Time course at a fixed position z. (b) A snapshot (fixed time t). The sense of rotation in (a) is opposite that in (b) because the traveling wave depends on $t - z/c$.

Circular polarization



Photon polarization

- As indicated earlier, light is characterized by a set of modes of **different frequencies, directions, and polarizations,** each occupied by a number of photons. For each monochromatic plane wave traveling in some direction, there **are two polarization modes**

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\mathbf{q}} A_{\mathbf{q}} U_{\mathbf{q}}(\mathbf{r}) \exp(j2\pi\nu_{\mathbf{q}} t)\, \widehat{\mathbf{e}}_{\mathbf{q}}.$$

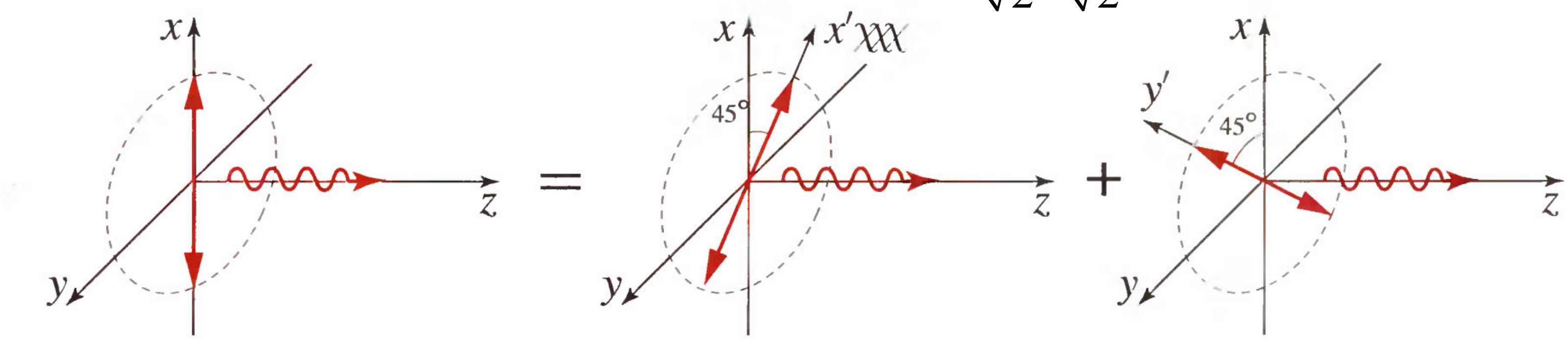
- Since the polarization modes of free space are **degenerate,** they are not unique. One may use modes with linear polarization in the x and y directions, linear polarization in two other orthogonal directions, say x' and y', or right- and left-circular polarizations.

$$A_{x'} = \frac{1}{\sqrt{2}}(A_x - A_y), \qquad A_{y'} = \frac{1}{\sqrt{2}}(A_x + A_y),$$

The components Ax, Ay are transformed from one coordinate system to another like ordinary Jones vectors, and the new components represent complex probability amplitudes in the new modes. ***Thus, a single photon may exist, probabilistically, in more than one mode.***

Photon polarization

- A photon in the x linear polarization mode is the same as a photon in a superposition of the x' linear polarization mode and the y' linear polarization mode with probability ½ each. $\textit{Jones vector}\,(1,0) \rightarrow (\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$



One x-polarized photon

One x'-polarized photon (probability ½)

One y'-polarized photon (probability ½)

A linearly polarized photon is equivalent to the superposition of a right- and a left-circularly polarized photon, ½ each with probability $\textit{Jones vector}\,(1,0) \rightarrow \frac{1}{\sqrt{2}}(1 \pm j)$

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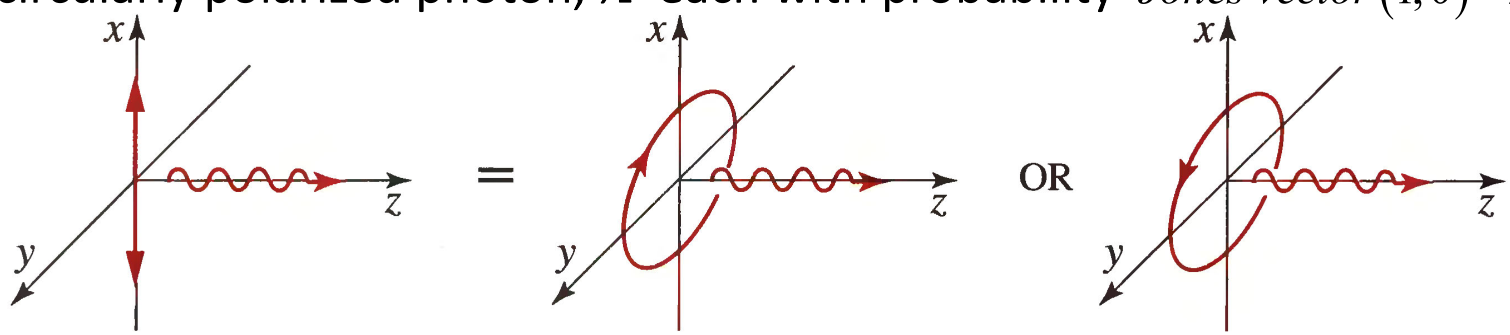


One LP photon

One RCP photon (probability ½)

One LCP photon (probability ½)

6 common examples of normalized Jones vectors.

Polarization	**Corresponding Jones vector**	
Linear polarized in the x-direction Typically called 'Horizontal'	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
Linear polarized in the y-direction Typically called 'Vertical'	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	
Linear polarized at 45° from the x-axis Typically called 'Diagonal' L+45	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	
Linear polarized at −45° from the x-axis Typically called 'Anti-Diagonal' L-45	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
Right Hand Circular Polarized Typically called RCP or RHCP	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}$	
Left Hand Circular Polarized Typically called LCP or LHCP	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$	

Jones matrices for common optical elements

Optical Element	Jones matrix
horizontal linear polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
vertical linear polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
linear polarizer at θ	$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$
quarter wave plate (fast axis vertical)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
quarter wave plate (fast axis horizontal)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

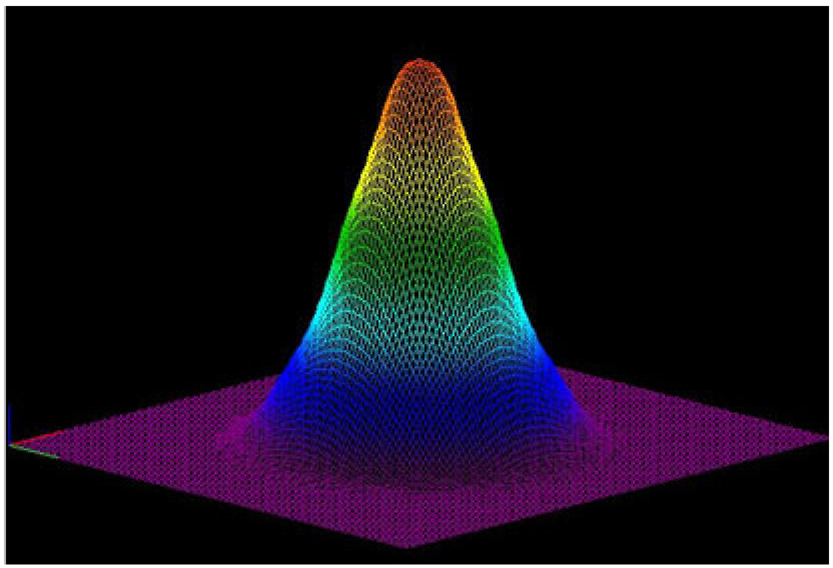
When light crosses an optical element the resulting polarization of the emerging light is found **by taking the product of the Jones matrix of the optical element and the Jones vector of the incident light**. Light which is randomly polarized, partially polarized, or incoherent must be treated using Mueller calculus.

Photon position

- When a photon impinges on a detector of small area **dA** located normal to the direction of propagation at the position r, its indivisibility causes it to be either **wholly detected or not detected at all**

The probability $p(\mathbf{r})dA$ of observing a photon at a point r within an incremental area dA, at any time, is proportional to the local optical intensity $I(\mathbf{r}) \propto |U(\mathbf{r})|^2$, so that

$$p(\mathbf{r})\,dA \propto I(\mathbf{r})\,dA.$$



$U(\mathbf{r})\ \exp(j2\pi\nu t)$

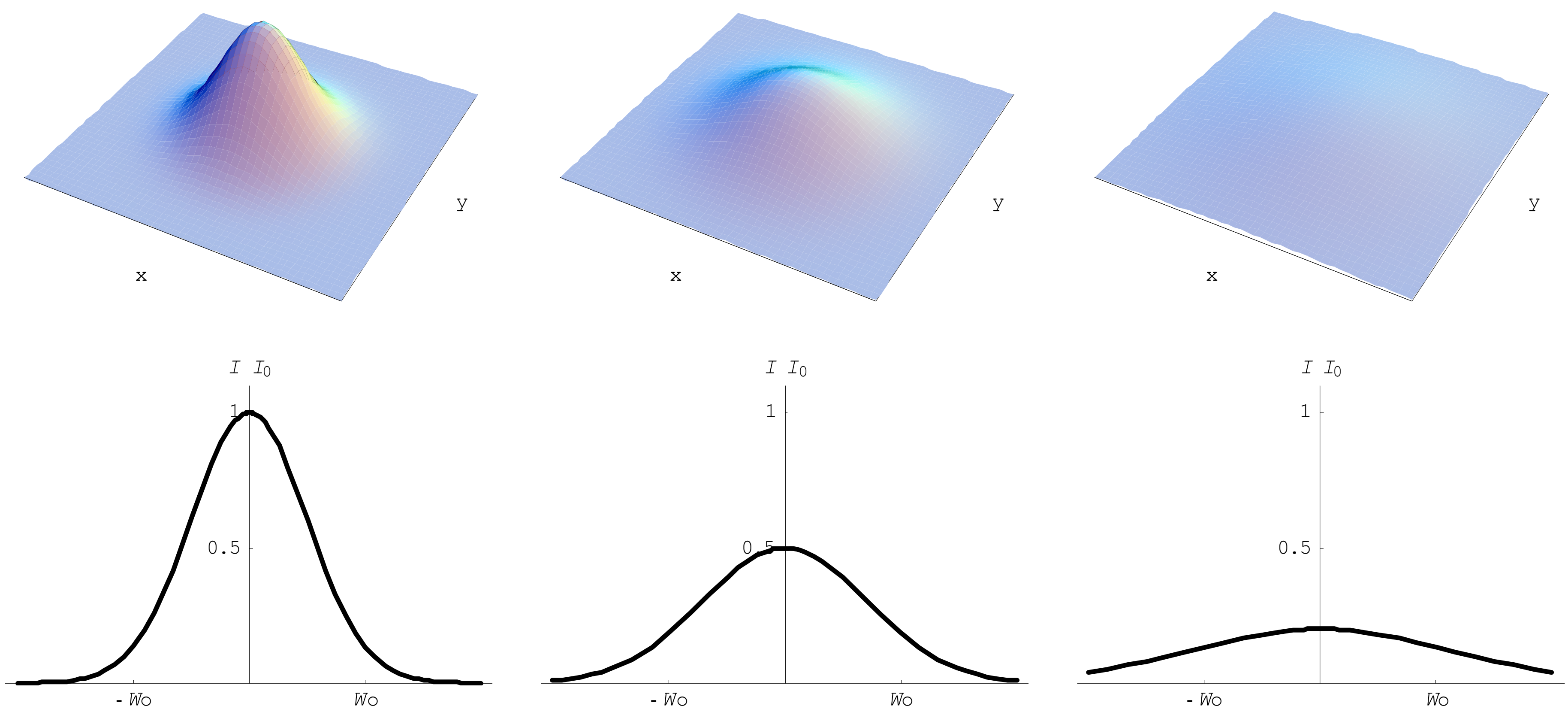
The photon is therefore **more likely to be found at those locations where the intensity is high.**

Optical photons behave as extended and localized entities. This behavior is called **wave particle duality**.

The localized nature of photons becomes evident when they are detected.

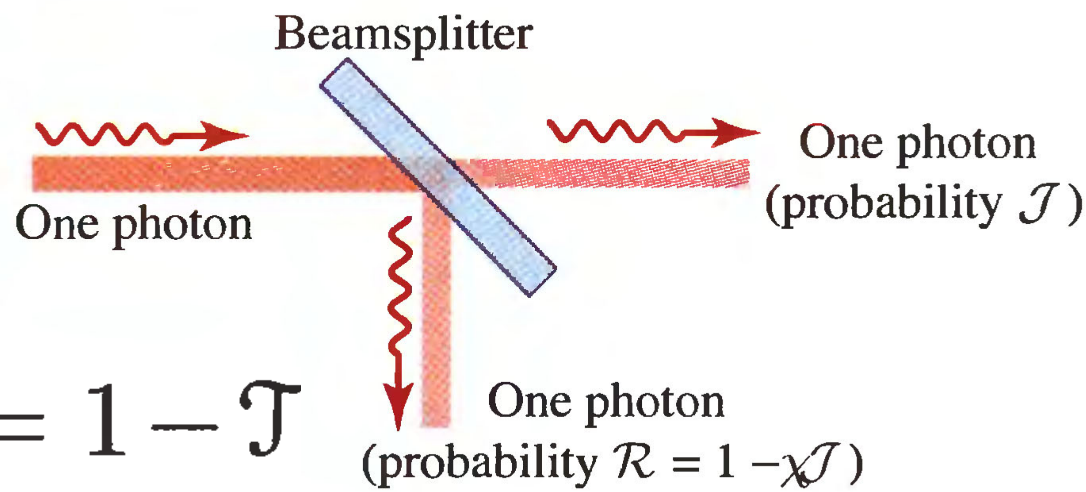
Gaussian Beam Intensity Plots

$$\frac{I(\rho,z)}{I_0}=\left[\frac{W_0}{W(z)}\right]^2\exp\left[-\frac{2\rho^2}{W^2(z)}\right]$$



Transmission of a Single Photon Through a Beamsplitter

- A photon is indivisible, it must choose between the two possible directions permitted by the beamsplitter.



intensity reflectance $\mathcal{R} = 1 - \mathcal{T}$

intensity transmittance $\mathcal{T}$

The probability that the photon is transmitted is proportional intensity transmittance

$$\mathcal{T} = I_t / I$$

The probability that it is reflected $1 - \mathcal{T} = I_r / I.$

Photon Momentum

In photon optics, the linear momentum of a photon is $\mathbf{p} = (E/c)\widehat{\mathbf{k}}$ where $E = \hbar\omega = \hbar c k$ is the photon energy.

- Photons carry momentum

$$\left|\vec{p}\right| = \frac{h}{\lambda}$$

- **Change in momentum corresponds** to the **force** and it can be calculated by the difference is **momentum flux S** between entering and leaving a object

$$\vec{F} = \frac{n}{c}\iint\left(\vec{S}_{in} - \vec{S}_{out}\right)dA$$

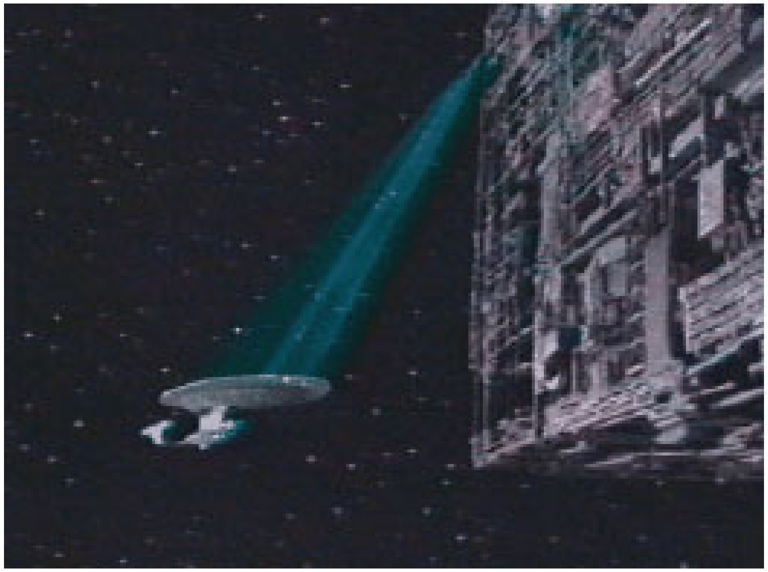
- Applying this formula to a 100% reflecting mirror reflecting a 60W lamp gives a pressure of:

$$\vec{F} = 2\frac{n}{c}\iint\left(\vec{S}_{in}\right)dA$$

$$\vec{F} = 2\frac{n}{c}W = 4\times10^{-7}\,N$$

Photon Momentum

- momentum associated with a photon can be transferred to objects of finite mass, giving rise to a force and causing mechanical motion. As an example, light beams can be used to deflect atomic beams traveling perpendicularly to the photons. The term radiation pressure is often used to describe this phenomenon (pressure force/area).
- .
- Sunlight on earth 0.5 nN/cm^2



Gravity pulls on a 1 kg mirror with 9.8 N so the force of the photons is negligible.

- However, if the same light is reflected by a object of 1 µg it can’t be ignored!

- Using a laser on a microscopic particle will realize this situation

Physics of optical trapping

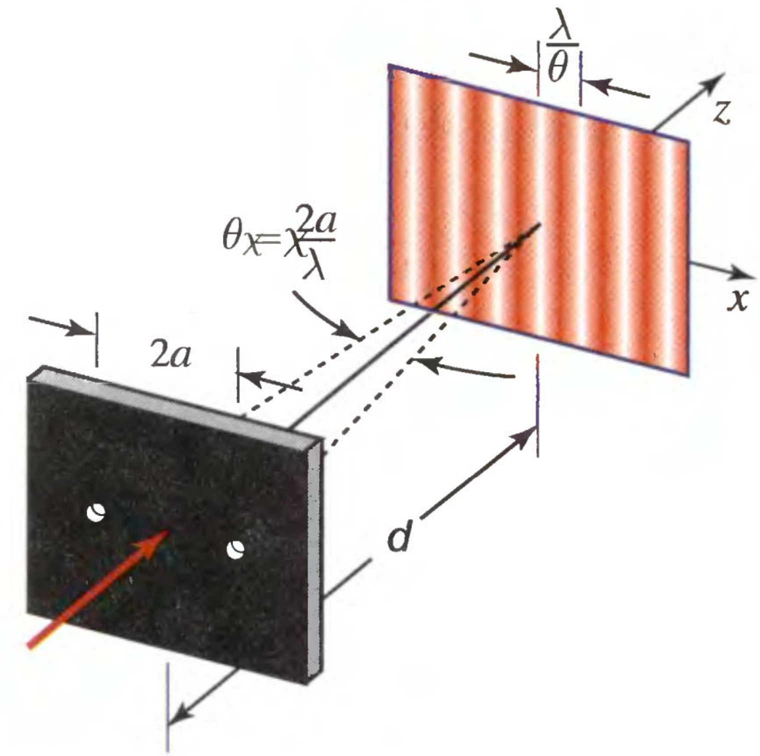
- The physics of the trapping mechanism is based on optical gradient and scattering forces arising from the interaction of strongly focused laser light with matter

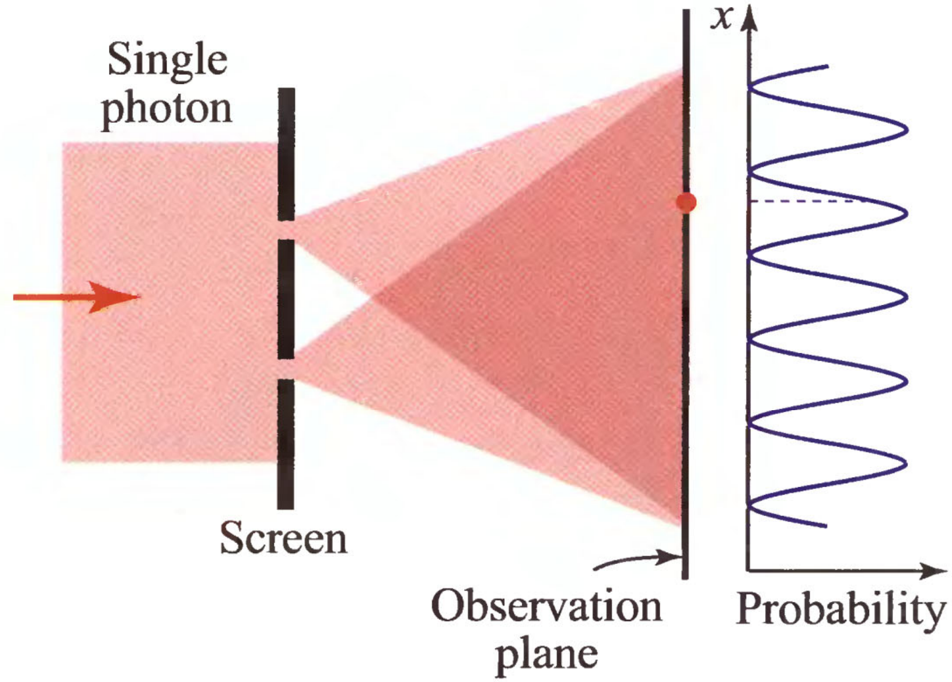
- Simple models that explain optical trapping behavior can be applied in the **Mie scattering (d >> λ)** and the **Rayleigh scattering (d << λ)** regimes depending on the size of the particle relative to the wavelength of laser light

- A real optical tweezers typically works in the intermediate (d ≈ λ) regime, requiring a rigorous application of complicated approaches such as Generalized Lorentz-Mie Scattering or T-Matrix theory (beyond the scope of this lecture!)

Photon Interference

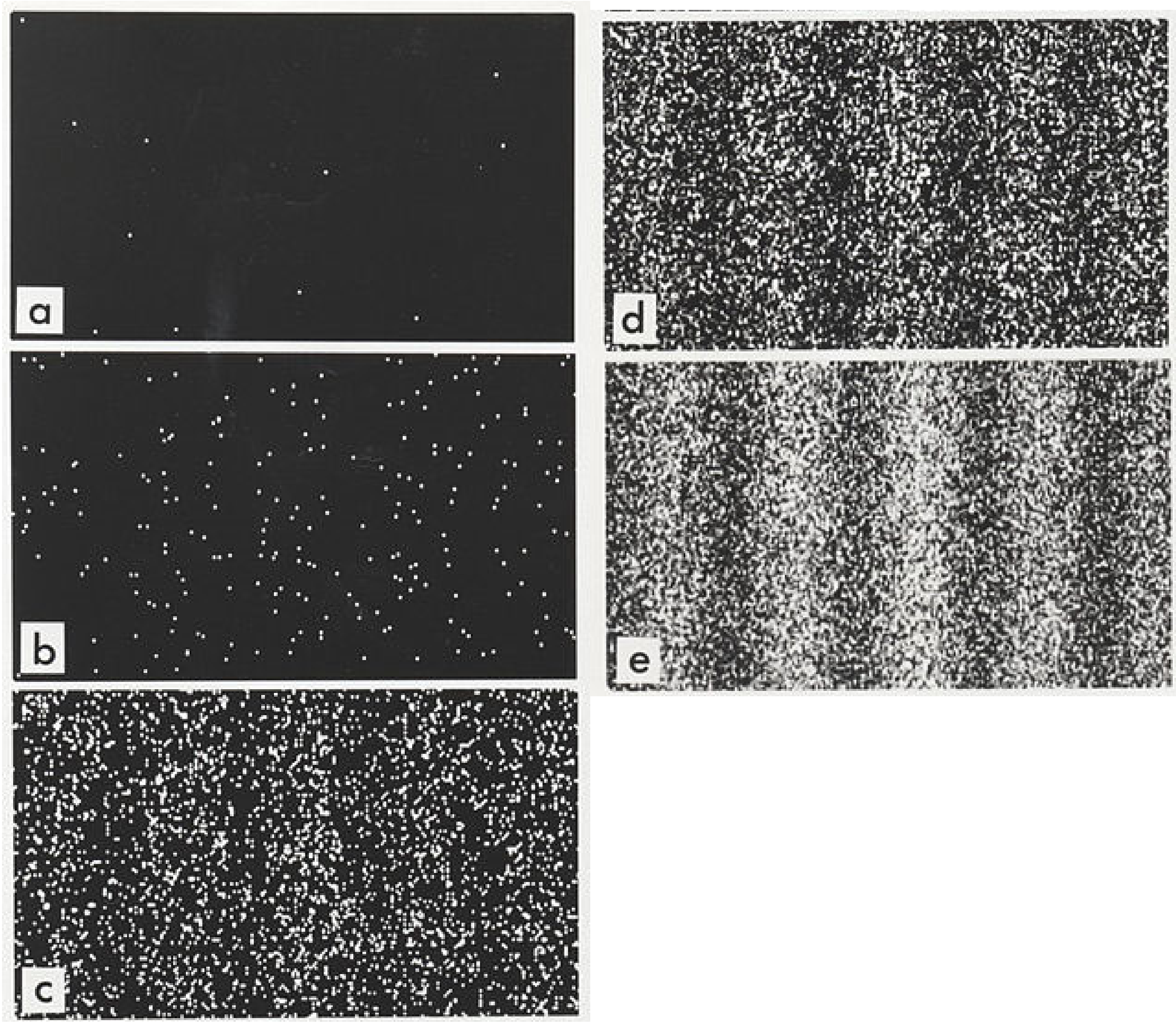
- Young's double-pinhole interference experiment is generally invoked to demonstrate the wave nature of light





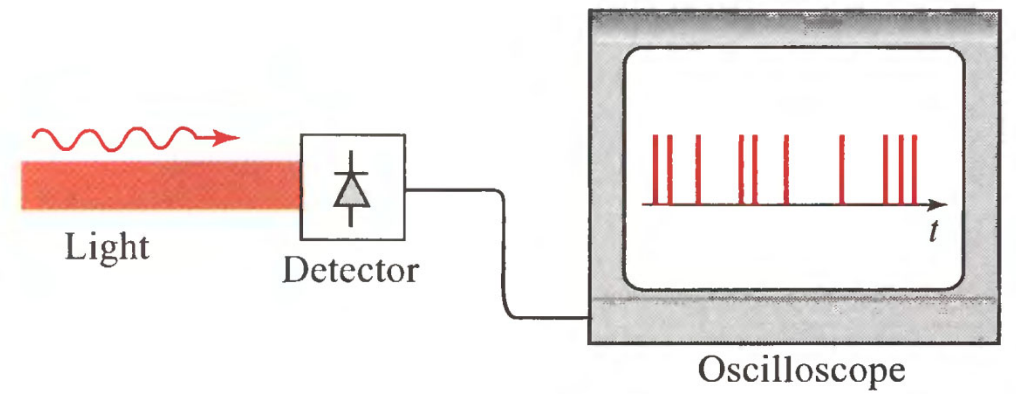
The occurrence of the interference results from the extended nature of the photon, which permits it to pass through both holes of the apparatus. This gives it knowledge of the entire geometry of the experiment when it reaches the observation plane, where it is detected as a single entity.

Photon Interference



Photon Streams

- The temporal pattern of such photon registrations can be highlighted by examining
- **the temporal and spatial behavior separately**. Consider the use of a detector with **good temporal resolution that integrates light over a finite area A**



$$P(t) = \int_A I(\mathbf{r}, t)\, dA.$$

Photon registrations at random localized instants of time for a detector that integrates light over an area A.

Mean Photon-Flux Density

Monochromatic light of frequency v and classical intensity I(r) (Watts/cm ^{2}) carries a mean photon-flux density

$$\phi(\mathbf{r}) = \frac{I(\mathbf{r})}{h\nu}.$$

Mean Photon Flux Density/ Mean Photon Flux

- Typical values of q; r for some common sources of light

Source	Mean Photon-Flux Density (photons/s-cm^2)
Starlight	10^6
Moonlight	10^8
Twilight	10^{10}
Indoor light	10^{12}
Sunlight	10^{14}
Laser light[a]	10^{22}

[a] A 10-mW He–Ne laser beam at $\lambda_o = 633$ nm focused to a 20-μm-diameter spot.

The mean photon flux Φ (units of photons /s) is obtained by integrating the mean photon-flux density over a specified area

$$\Phi = \int_A \phi(\mathbf{r})\, dA = \frac{P}{h\overline{\nu}},$$

As an example, 1 nW of optical power, at a wavelength λ 200 nm, delivers to an object an average photon flux Φ =10 ^{9} photons per second.

Mean Number of photons

- The mean number of photons **n** detected in the area A and in the time interval T is obtained by multiplying the mean photon flux Φ in by the time duration

$$\overline{n} = \Phi T = \frac{E}{h\overline{\nu}},$$

Classical		Quantum	
Optical intensity	$I(\mathbf{r})$	Photon-flux density	$\phi(\mathbf{r}) = I(\mathbf{r})/h\overline{\nu}$
Optical power	P	Photon flux	$\Phi = P/h\overline{\nu}$
Optical energy	E	Photon number	$\overline{n} = E/h\overline{\nu}$

Time Varying Light

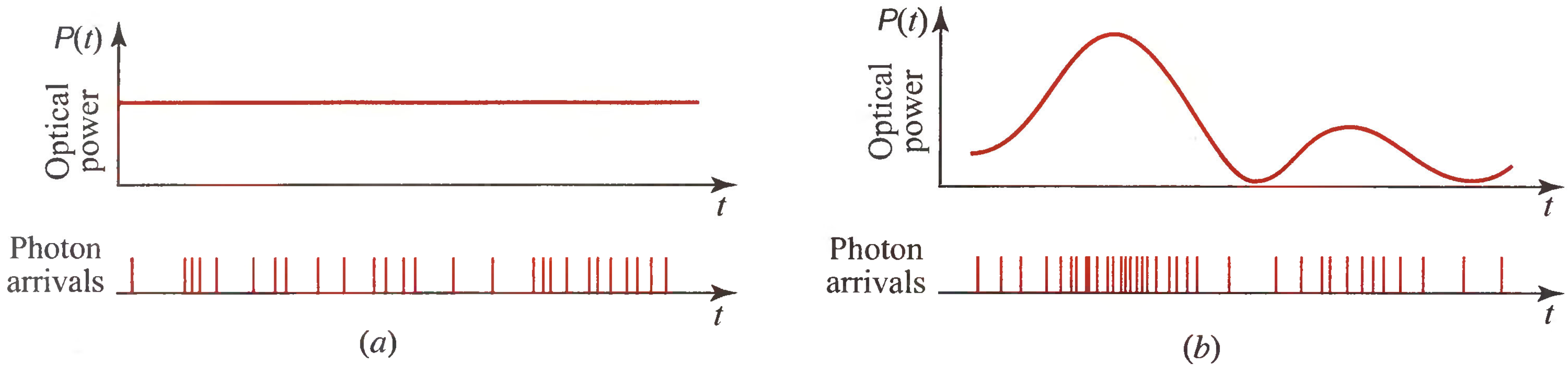
$$\phi(\mathbf{r}, t) = \frac{I(\mathbf{r}, t)}{h\overline{\nu}}.$$

$$\Phi(t) = \int_A \phi(\mathbf{r}, t)\, dA = \frac{P(t)}{h\overline{\nu}}, \quad \overline{n} = \int_0^T \Phi(t)\, dt = \frac{E}{h\overline{\nu}},$$

Randomness of Photon Flow

- For photon streams, the classical intensity **I(r,t)** determines the mean photon- flux density **ϕ(r,t).** The properties of the light source determine the fluctuations

- the times at which the photons are detected are random, their statistical behavior determined by the source,



(a)   Constant optical power and the corresponding **random photon arrival times**.

(b) Time-varying optical power and the corresponding random photon arrival times

Randomness of Photon Flow

- An understanding of photon-number statistics is important for applications such as reducing noise in weak images and optimizing optical information transmission.

- **Coherent light** has a constant optical power P. The corresponding mean photon flux Φ=P/hν (photons/s) is also constant, but the actual times of registration of the photons are random**. An expression for the probability distribution p(n) can be derived under the assumption that the registrations of photons are statistically independent. The result is the Poisson distribution**

$$p(n) = \frac{\overline{n}^n \exp(-\overline{n})}{n!}, \qquad n = 0, 1, 2, \ldots$$

- It is not difficult to show in and that the mean of the Poisson distribution is indeed n and its variance is equal to its mean:

$$\sigma_n^2 = \overline{n}.$$

Signal to Noise Ratio

- **The randomness of the number of photons** constitutes a fundamental source of noise that we have to account for when using light to transmit a signal. Representing the mean of the signal as **n,** and its noise by **the root mean square value is** σn, a useful measure of the performance of light as an information-carrying medium is the signal-to-noise ratio (SNR). The SNR of the random number n is defined as

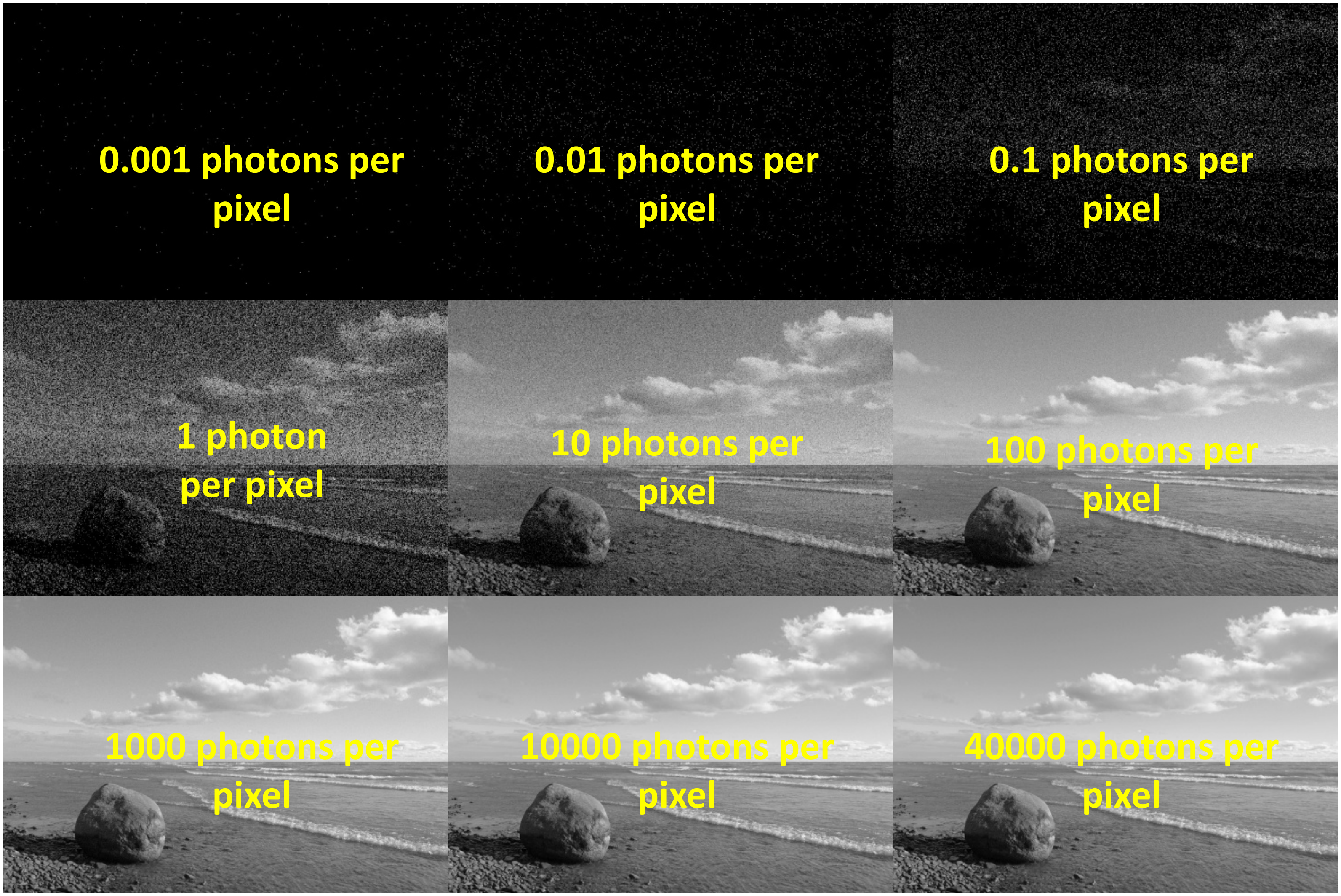
$$\mathrm{SNR} = \frac{(\text{mean})^2}{\text{variance}} = \frac{\bar{n}^2}{\sigma_n^2}.$$

For the Poisson distribution

$$\mathrm{SNR} = \bar{n},$$

so that the signal-to-noise ratio **increases linearly with the mean number of photon counts.**

Photon Shot Noise/ Exposure dependent



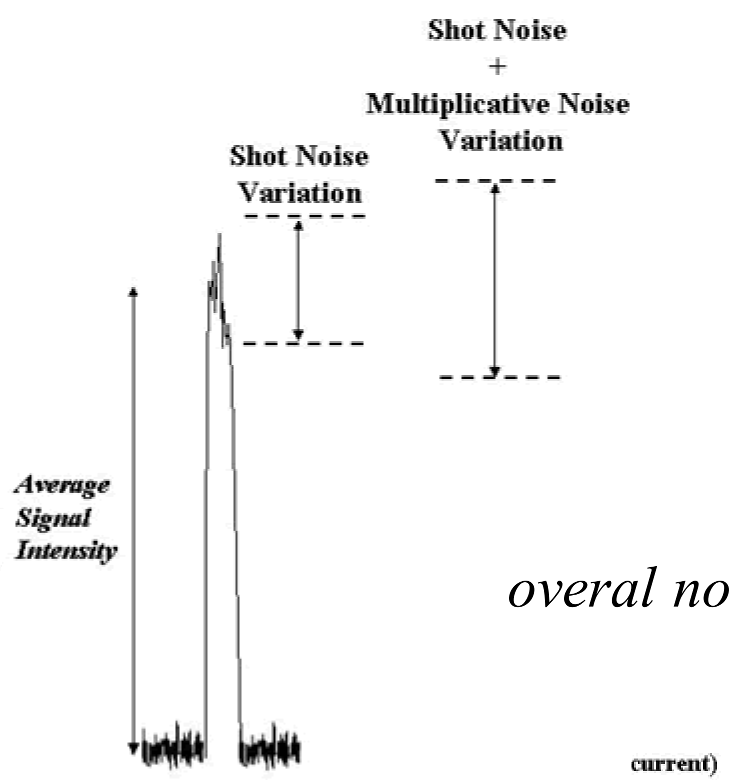
A photon noise simulation, using a sample image as a source and a per-pixel Poisson process to model an otherwise perfect camera (quantum efficiency = 1, no read-noise, no thermal noise, etc).

Noise Sources of a Detector

Photon Shot Noise – Counting statistics of the signal photons

-Originates from the Poisson distribution of signal photons as a function of time

-Random arrival of photons and electron is governed by Poisson distribution



- Dark Current Noise – Counting statistics of spontaneous electron generated in the device
- Johnson Noise – Thermally induced current in the transimpedence amplifier

$$overal\ noise = \sqrt{(readnoise)^2 + (darknoise)^2 + (shotnoise)^2}$$

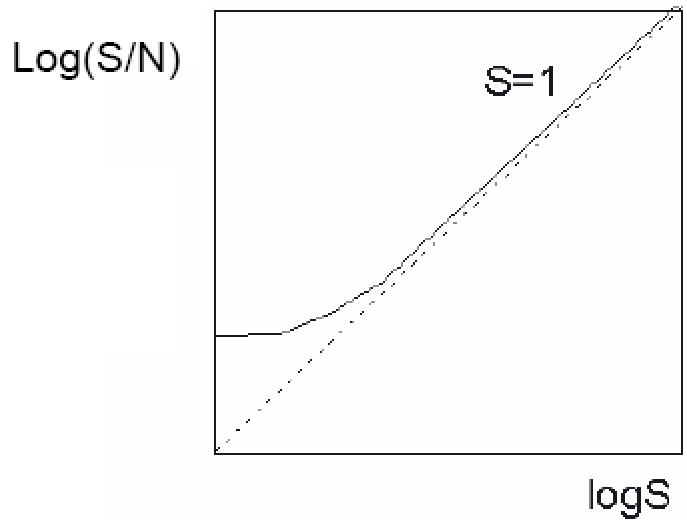
Photon Shot Noise

- Shot noise is white noise, just like Johnson noise. Does not exist unless current is driven through the device.
- This is termed **“white noise”** why?
- Because, like in white light**, all *frequencies* are equally represented**

- Standard deviation (or noise) and the photon noise limited Signal-to-Noise-Ratio (SNR) associated with detecting a mean of ‘N’ photons are given by

$$Noise\,(photon) \approx \sqrt{N}$$

$$SNR(photon) = \frac{Signal}{Noise} \approx \frac{N}{\sqrt{N}} = \sqrt{N}$$



- Means to **enhance** S/N
  - – Signal averaging: internally or externally
  - – Signal smoothing: boxcar averaging, moving average, polynomial smoothing (**keyword: convolution**)
  - – Filtering in the frequency domain: